# Inertia in the Brazil nut problem

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The rise dynamics of a large particle, in a granular bed under vertical vibrations, is experimentally studied with an inductive device designed to track the particle while it climbs through the granulate under different conditions. A model based on energy considerations is presented to explain our experimental data, drawing the important conclusion that it is the inertia of the particle, assisted by Reynolds dilatancy, the driven force behind its ascension mechanism. The ascension reveals a friction profile within the column which remains unchanged for different accelerations.

Industries dealing with nonconsolidated granular materials are aware of the following phenomenon: large particles in a bed of small ones rise and segregate to the top when the system is vertically shaken. This is an important issue, since very often granular mixtures lose their homogeneity properties when they are in vibrating environments (see, for instance, a recent review by Rosato and co-workers [1]). When particle separation in granular beds is the aim of an industrial process, the phenomenon can be used in our favor. In any case, a complete understanding of this problem is a scientific challenge not completely fulfilled despite the great number of experimental and theoretical studies carried out in the last years [2–6].

One of the first quantitative attempts to understand granular segregation using dynamic simulations was reported in 1987 [5]. The problem, baptized by Rosato and co-workers with the appealing name of "Brazil nut problem" (BNP), has been theoretically studied ever since, using several computer models [4,7–9]. Such studies focused mainly on the influence of size, friction, and excitation parameters. Some of these results support the hypothesis that it is reorganization or "void filling" beneath large particles, the universal mechanism promoting their upward movement through the bed. However, experimental studies find evidence that global convection rather than reorganization could be the driven force behind the BNP [6]. The dilemma is not yet settled as one can see in very recent reports [1,10]. Möbius *et al.* carried out an interesting experiment in which large intruders, of different densities, rise to the top of a vibrating granular column [10]. Two important aspects they observed are the following: air is important in the transport of the intruders through the bed, and decreasing the density of the intruder does not necessarily mean a monotonic increasing of the rise time, as might be suggested by studies in three dimensions (3D) [2] and 2D [11]. A new experimental work studying the influence of air drag in granular segregation has recently appeared [12], in which suppression of the segregation phenomenon has been observed if air is completely removed from fine binary glass-bronze mixtures of equally sized spheres (90–120 $\mu$m).

The aim of this paper is to report an experimental study that might contribute to the understanding of the physical mechanisms behind the BNP. With the exemption of few experimental studies [6,11], the experiments devised so far to study the BNP have been centered in measuring rise times of an intruder that climbs through a vibrated granular column. These measurements are important to describe the physics of the problem, but as we will see below they do not give a complete picture of it.

A magnetic technique is proposed here to track the movement of a metallic bead in a granular column under vertical vibrations. Not only rise times are measured, but the complete dynamics of the bead is obtained.

In Fig. 1 a schematic of the experimental setup is shown. A cylindrical column is made of plexiglas, with inner diam-
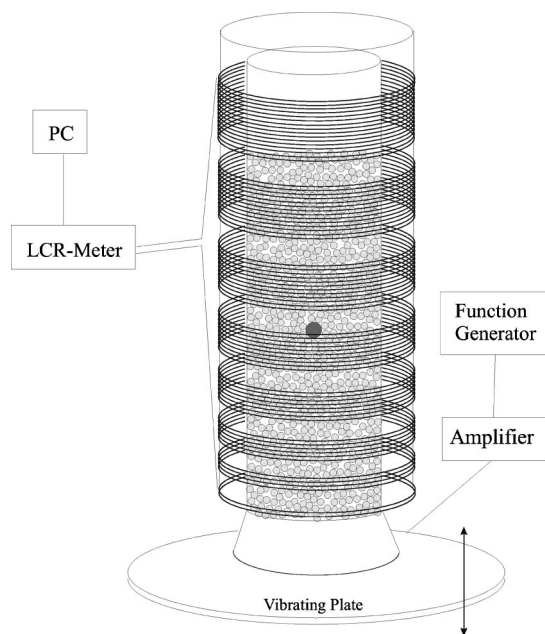


FIG. 1. A schematic of the experimental setup. A solenoid with a nonuniform number of turns per unit length is held concentrically to a vibrating column full of small particles. A larger metallic particle inside the column drifts up due to the BNP effect. At each vertical position, the climbing bead feels the magnetic field gradient inside the solenoid, changing accordingly the value of the solenoid inductance.

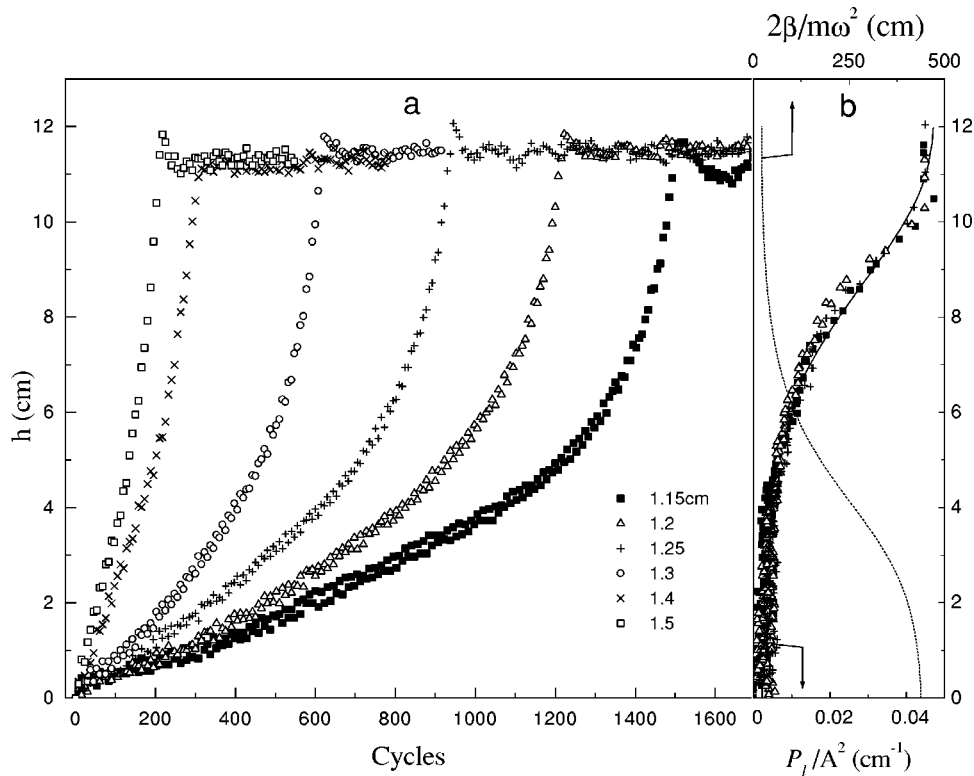*Corresponding author. Electronic address: cruiz@mda.cinvestav.mx

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FIG. 2. (a) Six experimental curves of $h$ vs oscillation cycles ($\omega t/2\pi$). Each curve represents the ascension dynamics of a bead in a granular column vibrated at different amplitudes: 1.15, 1.20, 1.25, 1.30, 1.40, and 1.50 cm. (b) The derivative of curves corresponding to $A = 1.15$, 1.20, and 1.25 cm divided by the square of $A$. The curves collapse into one, where the solid line is only a guide to the eye. The dashed line is the inverse of the solid one and is proportional to the friction force.

eter of 2.5 cm and 20 cm length. The plexiglas tube is fixed to a vibrating table. This table is fed with an amplified periodic voltage coming from a function generator (HP-33120A). On the outer surface of a wider plexiglas cylinder tube, a solenoid is made with a nonuniform number of turns per unit length. The first segment of the solenoid is formed by two loops, an empty space of 8 mm is left and a segment of four loops follows. A third segment is made with six turns and is also separated by the same distance from the previous one. The array continues in this fashion until the last segment of the solenoid has 28 turns. From the first segment to the last, there is a total length of 21 cm. The separation between the end of each segment to the beginning of the next one is always 8 mm. This nonuniform cylindrical solenoid is made with a thin copper wire. Finally, the solenoid is held from above concentrically to the granular column, without touching the vibrating table and the column.

In a typical experiment, the column is filled with small seeds or glass beads. Before pouring the beads into the column, a large sphere is put at the bottom. If there is some magnetic contrast between the bead and the granulate [13], the inductance of the solenoid will change as a function of the bead position $h$. The inductance $L$ of the solenoid is measured by a HP4284 LCR-meter at a frequency of 10 KHz. What we measure is the effective value of $L$, but $L$ changes according to where the particle is in a given moment (for instance, at the beginning of the experiment the sphere is at the center of the first solenoid segment and therefore, the inductance has its lowest value). A previous calibration of the instrument is manually done by measuring $L$ as a function of $h$ [obtaining after a polynomial fit an almost linear smooth function $L = f(h)$]. Thereafter, the experimental data $L$ vs $t$ obtained in a run are converted into $h$ vs $t$. With the

device and procedure just described, we are able to continuously follow the ascent of the large particle in the granulate.

Our first experiment measured the dynamics of a spherical steel bead in a granular column of nearly monodisperse cabbage seeds as a function of the vibrational amplitude. The diameters of the bead and the small particles are, respectively, 6.32 and 2 mm (their densities are, respectively, 7.8 and 1.1 g/cm$^3$). We maintain fixed the frequency of the vibration at 7.5 Hz. The amplitudes are measured optically using a laser diode, a flat mirror mounted on the vibration table, and a screen 1 m away from the table. Before each run, the metal bead is held at the bottom of the column magnetically while the system is shaken and compaction of the bed is obtained. Once the column compacts to an equilibrium height, the ball bearing is released and the experiment begins. Normally, the granular column becomes electrically charged after several runs, thus, in order to avoid this problem, each run is made with new and fresh seeds. Excellent reproducibility of the data is obtained as long as temperature and humidity do not change during the experiments.

Six experimental curves of $h$ vs number of cycles ($t\omega/2\pi$), for different vibration amplitudes (frequency fixed), are shown in Fig. 2(a). As we can note, the dynamics of the bead is strongly affected by the vibration amplitude. Once the bead has climbed the entire column, one can see that a little oscillatory reentrance occurs (see data points around 11 cm of height). The aspect ratio of the column (height divided by width) is important in order to produce a more pronounced reentrance (or whale [2]) effect. This point will be discussed later.

A quantitative interpretation of the experimental results shown in Fig. 2(a) certainly requires a theoretical model. Let us first propose the following physical picture of what we

believe occurs during the ascension of the bead in the granulate: we define time equal zero, the time when the entire system (bottom plate, walls, bead, and granulate) has, in its sinusoidal motion, a maximum upward velocity ($A\omega$) and zero acceleration. Time runs and the system starts to decrease the velocity due to its increasing negative acceleration. Before the acceleration of the system reaches the value of $-g$, the column behaves as a normal silo resting on the ground [14]. However, just when $a=-g$ the bead and granulate lose contact with the bottom plate of the cell. Due to wall friction, interparticle interactions, and the relative acceleration between the cell and the granulate, stresses reorganize and arches invert, from silolike shapes (inverted $V$'s) to $V$ shapes [15]. Such phenomenon has been observed previously in two-dimensional vibrating piles [16–18]. Moreover, due to this instantaneous stress reorganization or Reynolds dilatancy, the granular bed soon adjusts to the same acceleration of the walls (wall friction is always present and shears the bed, dragging it downwards). If the walls of the cell had no friction, once the negative acceleration of the cell reached and surpassed the value of $-g$, the entire granulate would lose contact with the bottom and travel freely as particles thrown upwards. In this condition, there would not exist a mechanism to delay or stop most of the bed particles and the intruder cannot climb through the column.

Although the intruder could feel the granular stress reorganization, due to its larger kinetic energy, it still follows a ballistic uprise, penetrating by inertia into the bed a small distance that we will call penetration length $P_l$. Inasmuch as we are plotting in Fig. 2(a) $h$ as a function of number of cycles (not time), it is clear that the derivative of any of these curves is precisely $P_l$ (not velocity).

The above physical picture suggests that on each cycle, the kinetic energy of the bead is lost by friction during its penetration into the granular bed. Therefore, a simple energy balance per cycle gives the following relationship:

$$1/2mv_{to}^2 = \beta(h)P_l, \tag{1}$$

where $v_{to}$ is the "take-off" velocity the bead has when the system reaches a negative acceleration $a=-g$, $m$ the mass of the bead, and $\beta(h)$ the friction force exerted upon it by the granulate. $v_{to}$ is simply the value of $\dot{z}(t)$ when $\ddot{z}(t)=-g$ [where $z(t)=A\sin(\omega t)$]. Thus,

$$v_{to} = [A^2\omega^2 - g^2/\omega^2]^{1/2}. \tag{2}$$

Equation (1) implies that most of the kinetic energy of the bead per cycle is dissipated by friction (indeed, the potential energy per cycle $mgP_l$ is negligible). Equation (1) can be tested by plotting $P_l$ as a function of $A$. Clearly, the parabolic behavior predicted by Eq. (1) is obtained, see Fig. 3.

This parabolic behavior holds at any height $h$ but in order to illustrate it more clearly we use the slopes of the lines shown in the inset of Fig. 3, which are linear fits to the lower parts of the ascension curves in Fig. 2(a).

Furthermore, since the left term of Eq. (1) is a constant depending only on the parameters of the vibration (does not depend on $h$), the right term must be a constant as well and,
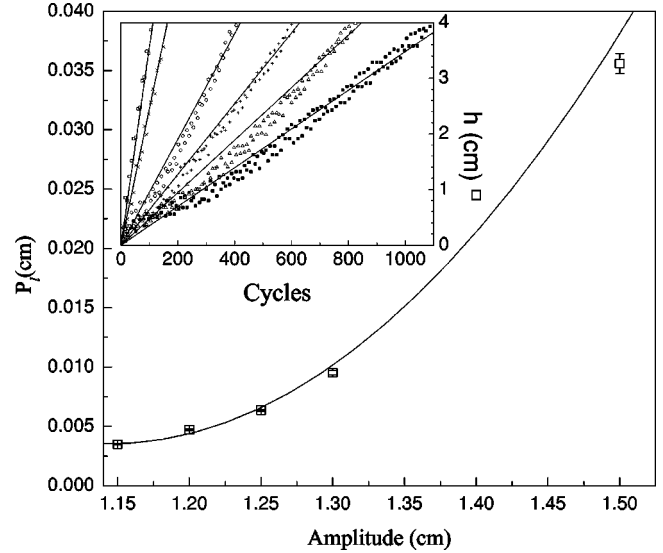


FIG. 3. $P_l$ as a function of vibrational amplitude $A$. The values of $P_l$ in this figure are the slopes of the lines shown in the inset, which represents the lower parts of the ascension curves shown in Fig. 2(a). The solid line is a parabolic fit put in the figure only as guide to the eye.

thus, $\beta(h)$ must be a function inversely proportional to $P_l$. In other words, the friction $\beta(h)$ the bead encounters along the column can be obtained directly by taking the inverse of $P_l$. Figure 2(b) shows $P_l/A^2$ as a function of $h$ for amplitudes 1.25, 1.20, and 1.15 cm. Surprisingly, the curves collapse into one, indicating that conservation of energy given by Eq. (1) might be considered the correct mechanism behind the BNP. This collapse also implies that the parabolic behavior of $P_l$ vs $A$ occurs at any $h$.

The three other curves, for $A$ equal to 1.30, 1.40, and 1.50 cm, are not shown here for the sake of clarity (they are increasingly noisy due to the smaller number of experimental points and the numerical derivative process, but behave the same way as the others as could be inferred by looking at Fig. 3). One can see by looking at Fig. 2(b) that the intruder feels the same friction profile during its ascension through the column at least for our acceleration conditions [$\Gamma = (2.5;3.4)$].

Equation (1) can be further tested in the following way: we investigate the ascension dynamics of a bead as a function of its own diameter. Four different bead diameters are considered: 4.74, 6.32, 9.46, and 11.10 mm. In each run, the acceleration of the vibration table remains fixed; the frequency being again 7.5 Hz and the amplitude of the vibrations 1.3 cm. The four beads have the same density as in the previous runs and the granular column the same height, 12 cm. Experimental curves of the ascension dynamics of the beads are not shown since they are similar to the ones presented in Fig. 2. However, plotting $P_l$ (evaluating the slopes the same way as in the inset of Fig. 3) as a function of bead diameter, we obtain the straight line shown in Fig. 4. This outcome is the expected behavior predicted by Eq. (1); $\beta$ being proportional to the cross section of the bead $D^2$ and the mass of the particle to $D^3$, one obtains $P_l \propto D$. As in the previous case, this linear behavior holds at any $h$.
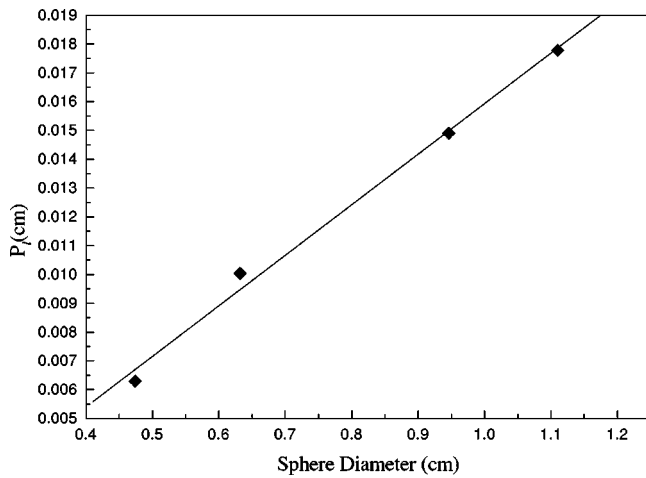
FIG. 4. Penetration length $P_l$ as a function of bead diameter.



FIG. 6. Oscillatory reentrance of the intruder into the granular bed caused by a convective cell on the top of the granulate for the widest cell.

If the underlying mechanism behind the BNP is related to stress-chain formation, which in turn is affected principally by the roughness of the internal walls and by their separation, then the ascension dynamics of the bead must change with $\sigma$ (cell diameter) and $\mu$ (wall friction coefficient). Figure 5 shows three experimental curves ($A$, $B$, and $C$) for the ascension dynamics of a bead in three granular columns with different diameters (5.3, 4.4, and 2.5 cm) under the same excitation conditions (frequency of 7.7 Hz and amplitude of 1.2 cm). Clearly, the wider is the container the faster the bead climbs through the granulate. In fact, arches are weaker (they span more distance) and therefore more easily disrupted by the ascending bead. A turning point would be the case when the separation of the walls is much larger. If they go to infinity (very large $\sigma$) Reynolds dilatancy on the bed caused by wall friction would become unimportant. The bed, to-
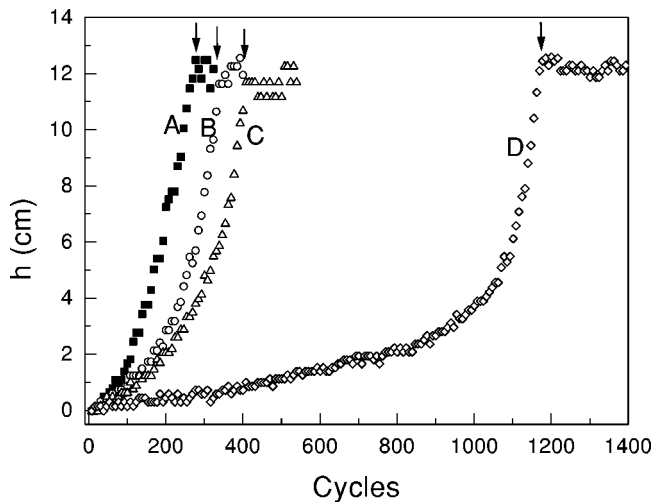


FIG. 5. Experimental curves ($A$, $B$, and $C$) representing the ascension dynamics of a bead in granular columns with different diameters: 5.3, 4.4, and 2.5 cm. Arrows indicate the rise time of the bead. Curve $D$ was obtained in a run using the column with the largest diameter (5.3 cm), but with a much greater wall friction coefficient.
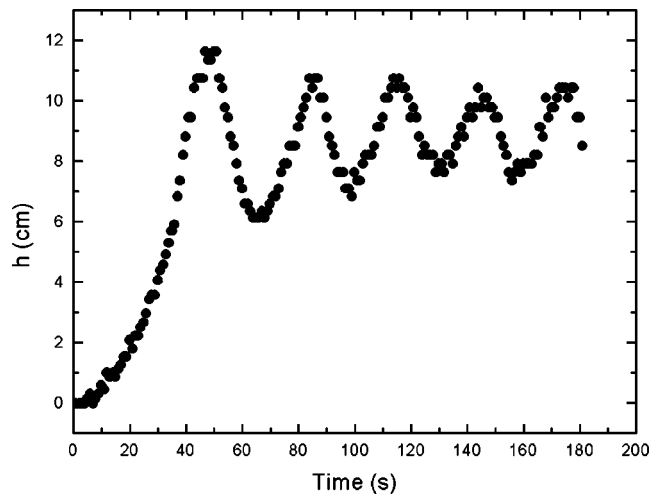
gether with the intruder, would move up and down imponderably and the intruder cannot ascend. To observe this condition with our experimental setup is not possible but is in agreement with the reported behavior in fine granular binary mixtures in vacuum by Burtally [12] and co-workers, in which the walls are hundreds of bead diameters away from each other. Indeed, they observe that air drives segregation and vacuum promotes mixing. In other words, air in this experiment is the mechanism to delay the lighter (kinetically poorer) phase of the mixture and segregation is seen. On the other hand, when air is evacuated, the kinetic energy contrast between the glass and brass spheres is no longer relevant and convection dominates, mixing both species.

Curve $D$ in Fig. 5 is obtained when the internal walls of container $A$ are covered by sandpaper. The higher the wall friction coefficient, the stronger the arch formation effect, and therefore the larger the rise time of the bead. This effect is surprisingly large if we compare curves $D$ and $A$.

We now come back to the discussion of the reentrance effect previously mentioned. As we said before, a very small (less than 1 cm) reentrance into the granulate is observed in some curves of Fig. 2(a). We also called the attention to the fact that the aspect ratio of the column is important to reduce or increase this whale effect. Thus, in order to study the reentrance of the intruder we use a wider column keeping the same height for the granulate (12 cm). Instead of 2.5 cm (as in Fig. 2), we use a 5.3-cm-diameter column. The ascension curve of the intruder in this case is curve $A$ shown in Fig. 5. Data taken beyond the rise time of the intruder (not shown in curve $A$ in Fig. 5) are depicted in Fig. 6. An oscillatory behavior is evident with a reentrance of about 6 cm. The intruder performs an oscillatory movement within a large convective cell formed in the upper part of the container as it may be expected. The formation of these convection cells has been attributed to wall friction and a granular temperature gradient along the column, and has been studied by nuclear magnetic resonance (NMR), dyed tracer particles, molecular dynamics simulations, and image velocimetry. In

some papers, the BNP is attributed to this convective motion [6,10]. As it can be seen in Fig. 6, the convection cell is strong enough to force the bead on top into the granulate again. However, since in our experiments we never observed an oscillatory movement with amplitude of 12 cm (the height of the bed), either the convection cell does not span the entire granulate or it does but is not able to drag the intruder through the entire column. Hence, in our conditions (where the intruder is positioned at the bottom of the granulate) convection cannot be the mechanism behind the BNP.

To confirm this, some convection experiments were carried out. We prepared tracer particles (black and red), put the red ones at the bottom of the column (a cylinder with inner diameter of 5.3 cm) and fill it with particles of the same size and density (the height of the granulate was 12 cm). We put afterwards the black tracer particles on top of the granular bed. First, we study the convection with no intruder (to avoid any disturbance of the possible convection cells). We tried different combinations of frequencies and amplitudes within the acceleration values reported above and the rise times of the red particles were carefully measured. In all our experiments the red particles diffused to the top very slowly, in no less than 20 min. The black particles went into shallow convective cells, down to 3–5 cm below the surface. Descending times were smaller but still large (few minutes). We did now, and separately, the experiment with different intruders (metal beads as above). The ascension times from the bottom to the top of the intruders were, for our accelerations, in any case no longer than 1 min. No matter what conditions we tried, the intruder climbed much faster than the tracer particles. We conclude that, in our conditions, the ascension of the intruder is not correlated with the convective cells. Indeed, in some experiments the intruders climbed through the granulate in 5 or 6 s, whereas tracer particles did it in 20 min in the same conditions.

Although full convection is not observed in narrow columns, one might think that a crossover between inertia and convection takes place for wider cells, where granular dragging in the upper part of the column could enhance the ascension velocity of an intruder.

In a very recent experiment [10], where only rise times of a bead climbing a granular bed are measured, the bead is positioned not in the bottom of the granulate, but very close to the top of the bed where a convective cell certainly exists. In that experiment it is seen that the rise time of the intruder is a nonmonotonic function of the density. This nonmonotonic behavior could be explained by the competition of two mechanisms: inertia and convection. For low intruder densities, convection seems to be dominant and the rise time drops when the intruder density decreases (the lighter the intruder is, the more easily it is dragged to the top). Inertia, however, becomes important when the intruder density increases and rise times drop again as inferred by our model. Here, it is clear that air acts as a lubricant. When the cell is evacuated, there is no lubricant and the drag caused by the convective flux is strong enough to carry the intruder upwards at the same speed regardless of its weight, observing the same rise time [10]. But, if the intruder starts its ascension from the bottom of the column (where there is no convection), the rise time of the intruder does depend on its density whether or not air is present. Very recently, it has been experimentally established in two-dimensional systems that rise times monotonically decrease with the mass of the intruder [11]. Our results agree with those 2D observations. Recently, we became aware of new results concerning the BNP which confirm also our findings [19].

We present experimental results that shed new light on the fascinating BNP. All previous experimental work carried out on this problem has mainly focused on measuring rise times, without measuring the complete nonlinear dynamics of the rising particle. Our gradient-coil technique solves this problem. Based on our data and a simple kinetic energy argument we conclude that in the BNP the bead climbs the granular column driven by inertia, assisted by Reynolds dilatancy through stress-chain formation. We used the velocity as the main parameter and not the dimensionless acceleration $\Gamma$ in order to stress out the role of the bead-bed kinetic energy contrast in the climbing mechanism.

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